Int. J. Heat Mass Transfer. Vol. 13, pp. 1153–1162. Pergamon Press 1970. Printed in Great Britain

THE THERMAL CONTACT RESISTANCE AT GOLD FOIL SURFACES

J. MØLGAARD* and W. W. SMELTZER

Department of Metallurgy and Materials Science, McMaster University, Hamilton, Ontario, Canada

(*Received* 16 *May* 1969 *and in revised form* 15 *December* 1969)

Abstract—The thermal contact resistance at gold foil surfaces has been measured over the pressure range 2·5 to 9 × 10^6 kg/m^2 and in the temperature range 50 to 300°C. Temperature had little effect on the thermal resistance, while the resistance was dependent on pressure in such a way as to suggest that both elastic and plastic deformation of the surface structure will occur in a manner determined by the sequence of pressure changes.

NOMENCLATURE

a,	radius of contact area at one hemispherical asperity, or half the width of contact at a semi-cylindrical asperity [*m*];
A,	total cross-sectional area [cm^2];
A_a,	area of contact at one asperity [cm^2];
A_r, A_t,	real area and total apparent areas of contact [cm^2];
C,	constant, equation (2) [thermal conductivity units];
E,	elastic modulus [kg/m^2];
F,	constant, equation (2) [°C^{-1}];
H,	total heat flow [W];
k_1, k_2, k_3,	dimensionless constants, equations (9), (10) and (13);
k_4,	surface geometry parameter, equation (14) [cm];
k_{Fe}, k_{PR},	thermal conductivity of Armco iron and Pt–13%Rh alloy;
L,	length [cm];
m, n,	exponents, equations (14) and (10) respectively [WdegK^{-1} cm^{-1}]
P,	pressure [kg/m^{-2}];
q,	exponent, equation (9);
Q,	heat flux [Wcm^{-2}];
r,	asperity radius [m];
R,	thermal contact resistance [degK cm^2 W^{-1}]
R_a,	constriction resistance [degK cm^2 W^{-1}]
R_G, R_i,	total thermal resistance of two gold foils and contact resistance at one interface [degK cm^2 W^{-1}];
R_0,	thermal resistivity [degKcm^3 W^{-1}];
T, T_k,	temperature, Celcius and absolute Kelvin;
T_5, T_6, T_7, T_8,	temperatures at thermocouples 5, 6, 7 and 8;
T_c, T_t,	temperature, lower surface and upper surface of sample gold foils;
W,	total load per asperity [kg] (hemispherical asperities), or load per unit length [kg/m] (semi-cylindrical asperities).

INTRODUCTION

WITH the development of an apparatus for the measurement of the heat transfer through oxide scales, it was necessary to determine the contact resistance at gold foils used to reduce the contact

---

* At present: Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St. John's, Newfoundland, Canada.

resistance between elements of the equipment. Whilst the values obtained for the contact resistance at gold foil surfaces were only corrections of a relatively minor kind, the results nevertheless have general interest. The use of indium foil for this purpose has been reported for temperatures ranging up to 116°C and pressures to 3×10^6 kg/m^2 since under these conditions its interfacial resistance was lower than for other materials and previous loading and temperature history had less effect [1]. Gold should have this value as an interfacial material at temperatures above the melting point of indium where oxidation of base metals can be significant. Accordingly, measurements of the thermal contact resistance at gold foil surfaces have been measured over relatively broad ranges of pressure and temperature in this investigation.

APPARATUS

The essential parts of the equipment mounted in a vacuum chamber to eliminate heat transfer by convection are illustrated in Fig. 1. The gold foils for which the contact resistance was measured were placed at position 10. This is also the location of the oxide sample, placed between the two gold foils or gold plated, for which the thermal conductivity is measured in the primary purpose of this equipment. Disc 1 supports these gold foils and conducts heat to them. This disc rests on a shaft, not shown, which extends through a sliding seal and out of the vacuum chamber. The upper end of this shaft contains a resistance heater. It is also surrounded by a small tube furnace. A hydraulic jack supplies the load applied to the bottom end of the shaft outside the vacuum chamber.

The gold foils are pressed upwards against the

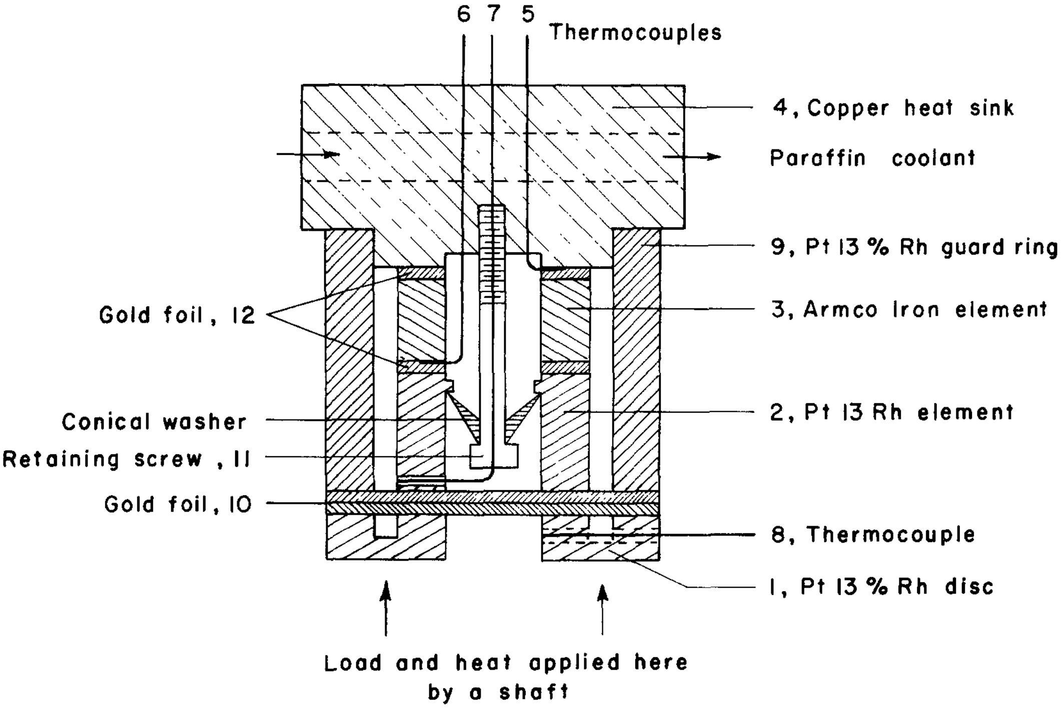

FIG. 1. Equipment essentials. Not to scale and omitting vacuum chamber, heaters and other auxiliary equipment.

essential core of the probe, a column consisting of two hollow cylinders, 2 and 3, of identical inside and outside diameters. Cylinder 2 and disc 1 are made of platinum–13% rhodium. Cylinder 3 is made of Armco iron and it is gold plated. This hollow cylindrical column is held together by the stainless steel retaining screw 11 and a stainless steel conical washer acting on the slight ledge at the upper end of the bore in element 2. This bolt and washer maintain the assembly in contact with the copper heat sink 4, through which a paraffin coolant is circulated. Elements 2 and 3 are surrounded by the guard ring 9 made of platinum–13% rhodium. The lower surfaces of element 2 and the guard ring 9 are ground and polished to be coplanar in order to provide equal pressure at their lower surfaces.

The heat flow through elements 2 and 3 is deduced by measuring the temperature drop through element 3 by means of thermocouples 5 and 6 at the upper and lower surfaces of this element. Each thermocouple is imbedded in gold foil at these positions. The thermal conductivity of Armco-iron is well established [2]. Since this metal could not be used in contact with a sample at position 10 due to the risk of oxidation, element 2 and disc 1 were constructed from a platinum alloy. The temperatures immediately above and below position 10 were measured by thermocouples positioned as close as possible to the surfaces, as indicated by 7 and 8.

A circular groove was cut in disc 1 so that its area in contact with the sample corresponded to that of the element 2. The o.d. and i.d. of this element are 0·78 and 0·54 cm respectively and its length is 0·60 cm. Element 3 which has the same diameters but a length of 0·256 cm, was gold-plated to prevent corrosion and reduce radiative heat losses. Thus, the equipment measured the heat flux through an annular area of the sample of 0·248 cm^2. The cross sectional area of the guard ring is about the same. For thick samples, the heat flow will be along curved paths, particularly at the edges of the annular contact area. A numerical relaxation analysis provided the corrections to apply in such cases, but such corrections were negligible with the thin gold foils in the present measurements.

Thermocouples 5 and 6 were chromel–alumel while thermocouples 7 and 8 were platinum vs. platinum–13% rhodium. The thermocouples were calibrated against a reference thermocouple, which in turn was calibrated at the National Research Council Laboratories in Ottawa. Thermocouples 5 and 6 were calibrated *in situ*. Since each of these thermocouples was imbedded in an interface between a gold foil and a neighbouring element, a mean temperature was obtained for a particular interface. Thermocouple 7 could not be calibrated *in situ* because heating would be required to a temperature higher than other parts of the assembly, particularly element 3, could safely be exposed to. We, therefore, compared the readings from thermocouples 7 and 8 using two different levels of heat flux through the equipment by varying the temperature of the paraffin coolant while maintaining the same mean temperature at the gold foils. By extrapolating to zero heat flux, one obtained the difference between these thermocouples. Such differences would be due to straining of the thermocouple wires during mounting.

The pressure on the samples could be varied between $2{\cdot}5 \times 10^6$–9×10^6 kg/m^2, the upper limit being determined by the load which the elements of the probe could safely carry without creep. The pressure could not be set to a greater accuracy than $\pm 0{\cdot}3$ kg/m^2, although the pressure could be reproduced to better than 0·2 kg/m^2.

PROBLEM ANALYSIS

The heat flow through the Armco iron element was calculated using the differential equation in an integrated form for heat flow through a cylinder

$$H = (C/F)(A/L)\exp(FT_6)\{1 - \exp[-F(T_6 - T_5)]\} \qquad (1)$$

where A = cross sectional area of the element, L = length, T_5 and T_6 are the temperatures indicated by thermocouples 5 and 6. F and C are constants in the equation,

$$K_{Fe} = C \exp(FT) \tag{2}$$

relating the thermal conductivity of Armco iron to the temperature T, on the Celcius scale, using the curve for recommended values [2]. These values fit equation (2) at least over a temperature range of 300°, which is more than the temperature difference across the iron element in our measurements. Each time equation (1) was used, the appropriate values of C and F were used.

The thermal conductivity of the platinum–13% rhodium alloy was obtained from experiments with this equipment [3]. This conductivity may be represented by

$$K_{PR} = 0{\cdot}607 - 0{\cdot}092(10^3/T_k) \tag{3}$$

where T_k is the absolute temperature, in degrees Kelvin. It was required in order to extrapolate from the temperatures indicated by thermocouples 7 and 8 to T_t, the temperature at the lower surface of element 2, and T_c, the temperature at the upper surface of disc 1.

The heat flux, Q, is given by

$$Q = H/A \tag{4}$$

where $A = 0{\cdot}248\ \text{cm}^2$, the cross-sectional area of elements 2 and 3. It is related to K_{PR} by

$$Q = \pm K_{PR}\frac{\mathrm{d}T_k}{\mathrm{d}x} \tag{5}$$

where the positive sign is used if T_k increases with x and the negative sign if T_K decreases with increasing x. Substituting (3) in (5) and integrating

$$\pm Qx = 0{\cdot}607\ T_k - 92 \ln T_k + \text{constant}. \tag{6}$$

When estimating T_c from T_8, the positive sign is used in (6) and $T_k = T_8$ when $x = 0$. Equation (6) then becomes

$$0{\cdot}607\ T_c 92 \ln T_c - (0{\cdot}607\ T_8 - 92 \ln T_8 - Qx) = 0 \tag{7}$$

where the distance between thermocouple 8 and the surface of element 2 is substituted for x. This equation was solved by iteration as part of the computer programme used in the processing of our measurements. A similar equation is used for obtaining T_t from T_7 by employing the negative sign of the left-hand side of equation (6) and integrating as before.

The total thermal resistance, R_G, of the gold foils at position 10 is given by equation (8)

$$R_G = (T_c - T_t)/Q \tag{8}$$

where Q is the heat flux through the foils. Since a sample had three interfaces, we assumed that the resistance at one interface, R_i, equals $R_G/3$. As mentioned above, the values of $T_c - T_t$ were corrected for the different thermoelectric coefficients of thermocouples 7 and 8. A simple calculation of the temperature drop across the gold foils only 0·0025 in. thick demonstrated that only 2 per cent of the measured T_c–T_t was due to a temperature drop across metal itself. Initial values of the total thermal resistance were used to correct for the resistance at 12 presented by the gold foil at each contact between the heat conductivity elements and the copper heat sink. These corrections made small changes in the heat flow calculations thus altering the deduced temperature T_c and T_t, in turn altering the calculated value of R_G. Final values of R_G were obtained by iterative repetition of the above procedure. The uncertainty in an estimation of R_G was due primarily to the uncertainty in estimating T_c–T_t. Accordingly, R_G or R_i has not been determined to better than ±25 per cent.

EXPERIMENTAL PROCEDURE AND RESULTS

The values of the contact resistance to be reported can only be regarded as approximate and highly dependent on the particular situation in which they were measured. They are reported not so much for the actual resistances obtained but for the inferences that can be drawn from the effects of load and temperature.

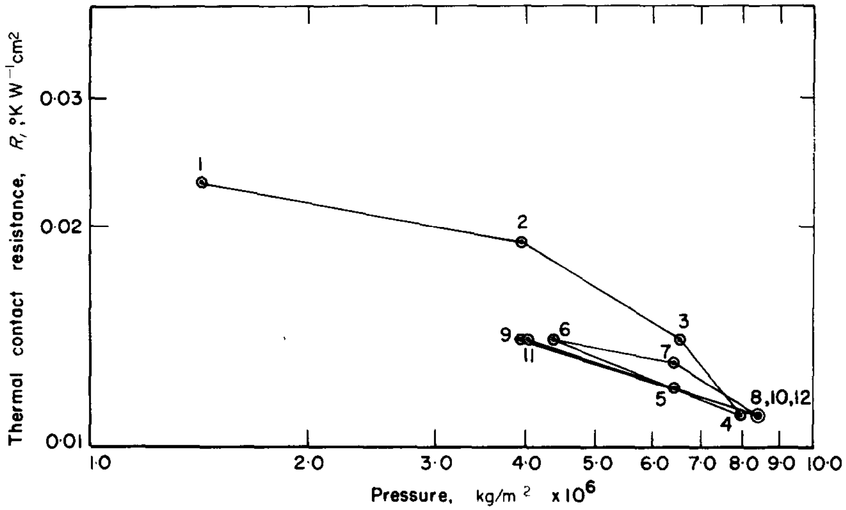

FIG. 2. Interfacial thermal resistance at one interface vs. pressure. Initial tests, starting at a low pressure, followed by cycling at high pressure. Sequence of measurements indicated by numbers. Sample temperature: 250°C.

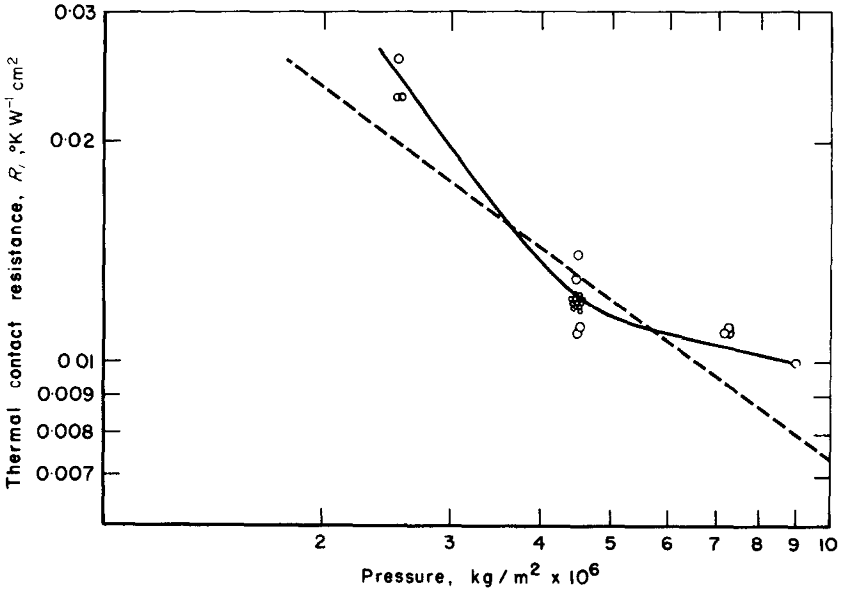

FIG. 3. Interfacial thermal resistance at one interface in the pressure range 2·5–10 × 10^6 kg/m^2 at 250°C. Tests started at 9 × 10^6 kg/m^2 and cycled at lower pressures ——— best fit to data, – – – – – regression line.

Initial series of measurements were carried out with the mean sample temperature at 250°C whilst the loading was initially 2·5 × 10^6 kg/m^2 and increased in steps up to 9 × 10^6 kg/m^2 and then cycled between this upper limit and 4·5 × 10^6 kg/m^2. These results are shown in Fig. 2, the sequence of the measurements being indicated by the numbers at the experimental points. It is clear that the initial loading produced a permanent reduction in thermal contact resistance, presumably by permanent deformation of the surface structure, but a large part of the contact resistance remains as a reversible function of pressure at pressures equal to or less than the maximum used.

As the gold foil mounted permanently in the equipment had already been subjected to the maximum load, the subsequent series of tests were done with fresh gold foil placed in position 10 and subjected initially to a pressure of 9 × 10^6 kg/m^2 at a temperature of 250°C and all subsequent measurements were done at lower pressures. These results are shown in Fig. 3. Here, reversibility of measurements was obtained to the lowest pressure of 2·5 × 10^6 kg/m^2. Also the absolute magnitudes of the contact resistance at each pressure in the range 4·5–9 × 10^6 kg/m^2 show good agreement in both measurement sequences depicted in Figs. 2 and 3.

A series of measurements is shown in Fig. 4 made with the same foils as in Fig. 3. For these measurements the pressure was maintained at 4·5 × 10^6 mg/m^2 while the temperature at the gold foil was varied from 50 to 300°C. The horizontal line gives the best least squares fit to the data while the vertical line indicates the standard deviation. It appears that the effect of temperature on the contact resistance was of minor importance, within the temperature range used and the accuracy obtained.

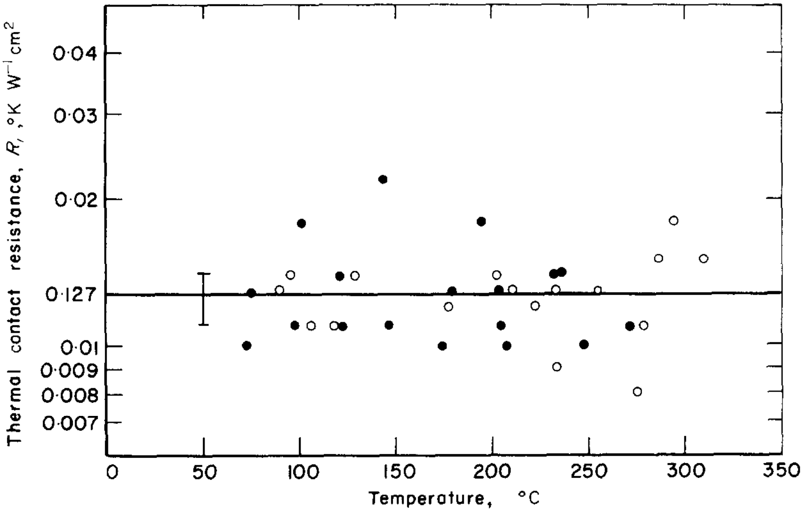

FIG. 4. Interfacial thermal resistance at one interface in the temperature range 50–300°C, at a pressure of 4·5 × 10^6 kg/m^2. Horizontal line = regression line. Vertical bar = standard deviation.

○ Measurements during ascending temperature.
● Measurements during descending temperature.

DISCUSSION

A value of the contact resistance across an interface depends on the thermal conductivities of the material on both sides of the interface, the total resistance being the sum of the resistance on each side of the interface. If the thermal conductivities on the two sides of the interface are very different, then the lower of the two thermal conductivities will have a greater influence on the total thermal resistance. Consequently the contact resistance at an interface between the platinum–13% rhodium alloy and gold would be greater than that between gold and gold. For this reason, the contact resistance at the platinum alloy–gold interface may be given to a better approximation by a value between $R_G/2$ and $R_G/3$ rather than the value of $R_G/3$ which is assumed in this investigation.

The values of the contact resistance, R_i, showed good agreement in both measurement sequences over the pressure ranges of reproducible behaviour. Our measurements extend to pressures three times higher than those previously used [1]. Where the pressure ranges overlap, our contact resistance measurements are about one-third of these earlier values. This is not unsatisfactory considering the scatter in both investigations. The difference could possibly be attributed to surfaces of different roughness and to the fact that the thermal conductivity of Pt–13% Rh alloy, in contact with the gold in our experiments, has a lower thermal conductivity than the iron employed in [1].

Contact between single elastic asperities of cylindrical or spherical shape and a flat surface is given by the Hertzian relationship [4],

$$a = k_1 (Wr/E)^q \tag{9}$$

where a is half the width of the contact area in the case of a cylindrical asperity or, alternatively, the radius of the contact area at a spherical asperity, r is the radius of the asperity, E is Young's modulus, W is the load per unit length for a cylindrical asperity or the total load for a spherical asperity and k_1 is a constant. $q = \frac{1}{2}$ for a cylindrical asperity; $q = \frac{1}{3}$ for a spherical asperity.

When extended surfaces in contact at a large number of asperities are considered, equation (9) leads often to a power law [5–7] of the form,

$$A_r/A_t = k_2 (P/E)^n \tag{10}$$

where A_r and A_t are the real area and the total apparent area of contact, respectively, k_2 is a constant, P is the pressure and n is an exponent which will depend on the surface structure.

If, for example, all the asperities are hemispherical and the same size and equally loaded, it follows from (9) that each contact area, A_a, is given by

$$A_a = \pi a^2 = \pi k_1^2 (Wr/E)^{2q} \tag{11}$$

i.e. comparing (10) and (11), we have $n = 2q$. If all the asperities are semi-cylinders, i.e. long ridges, of equal size and equally loaded, we have $n = q$. For any shape of asperity which is plastically deformed, the area of contact is proportional to the applied load, i.e. $n = 1$.

The real area of contact in the case of the gold foils at the highest pressure of 9×10^6 kg/m^2 would be of the order of 50 per cent of the total apparent area, decreasing to around 13 per cent at the lowest pressure of $2{\cdot}5 \times 10^6$ kg/m^2 if the deformation of surface asperities on the gold foil is assumed to be plastic and the hardness of gold at 205°C is approximately 20 kg/m^2. On the other hand, when elastic deformation is important, the proportion of the total apparent area of contact which will be in real contact can be much larger, since every region involving plastic deformation will be surrounded by an area involving elastic deformation.

The resistance presented to heat flowing through a small circular contact area is given by [8]

$$R_a = R_0/2a \tag{12}$$

where R_a is the resistance to heat flow due to the constriction presented by the small contact area, R_0 is the resistivity of the material assumed to be the same on both sides of the contact, a is the radius of the contact area. In view of equation (7),

$$R_a = k_3 R_0 A_a^{-\frac{1}{2}} \tag{13}$$

for individual asperity contacts.

In general, we would expect from (10) to (13) that the relationship between total contact resistance and pressure for two surfaces in contact would be of the form,

$$R = k_4 R_0 (P/E)^{-m}. \tag{14}$$

When all the contact areas are circular and of equal size, it follows from (13) that $m = n/2$. For long contacts as at ridges, for which the width of the contact area rather than the length varies with load, m will be close to n. If the surface topography is such that progressively more circular contact regions are formed as the contact load is increased m will be between n and $n/2$ [5]. It is apparent from equations (10) and (11) that the values of n for a distribution of asperity shapes and heights will be greater than one-half for semi-cylindrical asperities or one-third for hemispherical ones. In the case of plastic deformation only, n will be unity. Thus, if m is known from experimental measurements of contact resistance, we can deduce that n is between m and $2m$. If m is less than one-half, the surface deformation involved in the contact is most likely elastic, though there may be some plastic deformation. If m is between one-half and unity, the deformation could be entirely plastic, though elastic deformation cannot be ruled out. If m is greater than unity, the deformation must be primarily elastic.

There is no theoretical reason for an upper limit to n with elastic deformation, although in practice n tends to be close or less than unity [9]. When n is greater than unity, the real area of contact may be increasing at a greater rate than the increase of pressure producing the area increase if every increment of pressure brings large number of additional asperities into contact. A more likely explanation is that increasing pressure gradually changes the contact topography from one of a number of isolated contact regions, well separated from each other in relation to the size of the individual areas, to a number of clusters or groups of contact regions, the individual contact areas within each cluster being close to each other. It has been shown [10] that the thermal resistance of such clusters is equal to that of a single contact area similar in size to that of the whole cluster. The transformation from isolated contact regions to clusters would be similar, in its effect on the contact resistance, to an increase in the total real contact area greater than the actual increase of real contact area.

An explanation of the effect of pressure on the variation of the thermal contact resistance at the gold foil surfaces would be as follows. In Fig. 2, the lines joining points 1, 2, 3 and 4 have a negative slope going from about $-0{\cdot}2$ to $-0{\cdot}8$, as the load is increased for the first time. The subsequent measurements in this series all lie close to the line of slope $-0{\cdot}36$. This would imply that the deformation of the surface asperities is initially elastic and that the points of contact are well separated. Further increase in load produces both plastic deformation and a grouping of the contact areas in clusters, giving m values greater than 0·5. Once the maximum load has been reached and the load is then reduced, the elastic strains surrounding the plastically deformed regions could destroy many of the contacts formed at the maximum pressure, even though the plastic deformation will have altered the unstressed shape of the asperities. When the load is cycled below the maximum, values of the contact resistance are obtained reversibly since the deformation would be primarily elastic [12].

In Fig. 3, the first recorded measurement was at $8{\cdot}9 \times 10^6$ kg/m^2. The dashed line, which indicates the best straight line fit, does not adequately represent the experimental results. A curve obviously is a better representation. Below 4·5 kg/m^2, the curve has a slope of about $-1{\cdot}3$; above 4·5 kg/m^2, the slope is $-0{\cdot}35$, in good agreement with the final reproducible slope in Fig. 2. These results are consistent with the reasoning that the surface deformation is primarily elastic, the large change in the contact resistance at pressures below 4·5 kg/m^2 being associated with a number of clusters of contact regions, the individual contact areas within each cluster being close to one another.

Any effect of temperature over the range 50–300°C on the contact resistance at constant pressure could not be detected, Fig. 4. These findings imply that the heat transfer across the interface is primarily by conduction rather than radiation. However, a detailed analysis cannot be carried out to give a definitive answer to the relative degree of heat transfer by radiation due to the accuracy obtained of the individual measurements.

CONCLUSIONS

Results have been presented for the thermal contact resistance at the surface of gold foils for a range of pressures and temperatures. In the pressure range of $2{\cdot}5 \times 10^6$–$9{\cdot}0 \times 10^6$ kg/m^2, bonding produced a permanent reduction in contact resistance associated with plastic deformation of surface structure and a reversible pressure variation in this resistance associated with elastic deformation. The minimum value of the thermal contact resistance was approximately $0{\cdot}01\,°\mathrm{KW}^{-1}\,\mathrm{cm}^2$. Its value at constant pressure was independent of temperature over the range 50–300°C which indicated that heat transfer across an interface was primarily by conduction.

ACKNOWLEDGEMENTS

This research was carried out under a grant from the American Iron and Steel Institute and the National Research Council of Canada. In addition, J. Mølgaard was indebted to the National Research Council for a post-doctoral research fellowship.

REFERENCES

1. J. P. Moore, T. G. Kollie, R. S. Graves and D. L. McElroy, Thermal conductivity measurements on solids between 20 and 150°C using a comparative-longitudinal apparatus, Oak Ridge National Lab. Report ORNL-4121 (1967).
2. R. W. Powell, C. Y. Ho and P. E. Liley, *Thermal Conductivity of Selected Materials*. National Bureau of Standards, Washington, Dept. NSRDS-NBS8, (1966).
3. J. Mølgaard and W. W. Smeltzer, The thermal conductivity of 87% platinum–13% rhodium alloy. *J. Less Common metals* **16**, 275–278 (1968).
4. S. Timoshenko and J. N. Goodier, *Theory of Elasticity*. McGraw-Hill, New York (1951).
5. J. F. Archard, Contact and rubbing of flat surfaces, *J. Appl. Phys.* **24**, 981–988 (1953).
6. A. Lodge and H. G. Howell, Friction of an elastic solid, *Proc. Phys. Soc. Lond.* **67B**, 89–97 (1954).
7. C. Rubenstein, A general theory of the surface friction of solids, *Proc. Phys. Soc. Lond.* **69B**, 921–933 (1956).
8. R. Holm, *Electric Contacts Handbook*, pp. 13–27. Springer-Verlag, Berlin (1958).
9. J. Mølgaard, A co-efficient of friction which increases with increasing load. Results with undrawn nylon monofilaments, and a theoretical explanation. *Proc. Phys. Soc. Lond.* **79**, 516–534 (1962).
10. J. A. Greenwood, Constriction resistance and the real area of contact, *Br. J. Appl. Phys.* **17**, 1621–1632 (1966).
11. R. Holm, *Electric Contacts Handbook*, pp. 47–48. Springer-Verlag, Berlin (1958).
12. F. P. Bowden and D Tabor, *The Friction and Lubrication of Solids*. Vol. 1, pp. 22–24. Oxford University Press (1950).

LA RÉSISTANCE DE CONTACT THERMIQUE À DES SURFACES DE FEUILLES D'OR

Résumé— La résistance thermique de contact à des surfaces de feuilles d'or a été mesurée dans la gamme de pressions de 24 à 88. 10^6 Pascals et dans la gamme de températures de 50 à 300°C. La température avait peu d'effet sur la résistance thermique, tandis que la résistance dépendait de la pression de telle façon qu'elle suggérait que la déformation à la fois élastique et plastique de la structure de la surface se produisait d'une façon déterminée par la succession des changements de pression.

DER THERMISCHE KONTAKTWIDERSTAND AN GOLDFOLIENFLÄCHEN

Zusammenfassung—Der thermische Kontaktwiderstand an Goldfolienflächen wurde im Druckbereich von 2·5 bis 9. 10^6 kg/m^2 und im Temperaturbereich von 50 bis 300°C gemessen. Die Temperatur hatte wenig Einfluss auf den thermischen Widerstand, wogegen der Widerstand vom Druck so abhing, dass sowohl eïne elastische als auch eine plastische Verformung der Oberflächensttuktur in der Weise angenommen werden konnte, wie sie den Druckwechseln entsprach.

ТЕПЛОВОЕ КОНТАКТНОЕ СОПРОТИВЛЕНИЕ НА ПОВЕРХНОСТЯХ ИЗ ЗОЛОТОЙ ФОЛЬГИ

Аннотация—Тепловое контактное сопротивление на поверхностях из золотой фольги измерялось в диапазоне изменения давления от 2,5 до 9.10^6 кг/м^2 и температуры от 50 до 300°С. Температура незначительно влияла на тепловое сопротивление, но оно зависит от давления при условии, что упругая и пластическая деформация происходят в зависимости от последовательности изменения давления.